# Cost Analysis of a Two-Unit Standby Industrial System with Varying Demand 

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#### Abstract

In this paper we develop a stochastic model for two-unit standby system by making one or both the units operative depending upon the load/demand. The system under consideration is assumed to have two shifts of working and whole system undergoes for scheduled preventive/corrective maintenance before starting the second shift. Mathematical formulation of the problem determining the transition probabilities of various states are developed considering two types of failure for each unit (Type -I which has no standby and the Type - II which has standby). Various reliability metrics such as MTSF, availability, busy period and profit have been discussed for measuring the system effectiveness by using semi-Markov process and regenerative point technique. The proposed model is finally applied to rice manufacturing plant as a case study.


Index Terms: Availability, Busy Period, MTSF, Regenerative point technique, Semi-Markov Process
Nakagawa, Kumar and Agarwal, Sridharan and Mohanavadivu $[4,6,10]$ studied the stochastic behaviour of a two-unit cold standby redundant system. Goel et al.[2] and Khaled et al.[3] also studied two unit standby systems. Goel
et al.[1] discussed the reliability analysis of a system with preventive maintenance and two types of repair. Rander et al.[8] studied a system with two types of repairmen with imperfect assistant repairman and perfect master repairman. Taneja et al.[11] collected the real data on failure and repair rates of 232 programmable logic controllers (PLC) and discussed reliability and profit analysis of a system which consists of one main unit (used for manufacturing) and two PLCs (used for controlling). Initially, one of the PLCs is operative and the other is hot standby. Tuteja et al.[12] discussed the cost benefit analysis of two server two unit warm standby system with different types of failure. The cost analysis of a two-unit cold standby system subject to degradation, inspection and priority has been analyzed by Kumar et al [5]. Shakuntla et al.[9] discussed the availability of a rice industry. Wang Z et al.[13] analyzed the reliability of systems with common cause failure under random load. Recently, Wu and Wu [7] studied reliability analysis of twounit cold standby repairable systems under poisson shocks.

In the research mentioned above on standby systems, it was found that their analysis for system reliability are based on the various hypothetical failure and repair situations and assumed numerical values. However, no satisfactory work has been carried out for varying demand of system in the field of reliability. There may be different situations depending upon demand/load. Incorporating this situation, we present a new contribution and motivation in the present paper to the reliability literature in terms of real case study of an industry in which a two-unit stand by system with varying demand has been analyzed. We have proposed a model which analyzes mean time to system failure (MTSF), availability of the system and cost benefit. This type of model can be applied to any industries where standby systems are used.

The proposed is finally applied to a rice manufacturing plant which converts paddy into rice.

This paper has been organised as follows: In Section 2 a brief description of a rice manufacturing system is presented. The various notations and assumptions of two-unit standby system are also discussed in this section. The mathematical formulation for stochastic model determining, transition probabilities and mean sojourn times, are developed in Section 3. This section also deals with the formulation of Mean Time to System Failure, busy period analysis. In Section 4, we discuss methodologies to compute various reliability metrics. Certain conclusions based on the present study are finally drawn in Section 5.

## 2. SYSTEM DESCRIPTIONS AND NOTATIONS

In this system, it is assumed that one or both the units are made operative depending on the demand. Each unit has two types of failure-one due to failure of the component having no standby which is referred as Type -I failure and the second due to failure of any other component having its standby referred as Type - II failure. The processing done on each of the components of the unit before the component with no standby is transferable to the corresponding component of the other unit. But the processing in pending due to failure is completed only on the concerned unit after that component is repaired. Various measures for the effectiveness of the system such as mean time to system failure (MTSF) and availability are obtained assuming exponential distribution for failure time and taking arbitrary distribution for repair times. The model thus developed will be applied to a rice manufacturing plant as a good example of the present system model.

This plant has two units which are made operative depending upon demand. Both the systems are of eight ton capacity. The system under investigation considers the situation where the system has two shifts of working and before starting the second shift the whole system undergoes for scheduled preventive/corrective maintenance. Either one or both the units of this system is made operative depending on the demand. As mentioned above each unit maintains two types of failures, Type-I failure is due to the component color sorter and Type-II failure is due to failure of any of the following components of unit i.e. paddy separator, husker, destoner, polisher. The processing done on each of the components of the unit before the component colour sorter is transferable to the corresponding component of the other unit but the processing pending due to failure of colour sorter is completed only on the concerned unit after it is repaired. The system is observed at suitable regenerative epochs by using regenerative point technique. As shown in Fig. 1 the states occurring in this bracket $\left\{S_{0}\left(B_{0}, B_{0}\right), S_{1}\left(B_{0}, B_{s}\right), S_{2}\left(B_{r}, B_{r}\right), S_{3}\left(B F_{r_{1}}, B_{0}\right), S_{4}\left(B F_{r_{2}}, B_{0}\right), S_{5}\left(B_{o p}, B_{0}\right)\right\}$ are regenerative states. However, the states which are failed and non-regenerative are presented in this bracket $\left\{S_{6}\left(B F_{R_{1}}, B F_{W R_{1}}\right), S_{7}\left(B F_{R_{1}}, B F_{W r_{1}}\right), S_{8}\left(B F_{R_{2}}, B F_{W r_{2}}\right), S_{9}\left(B F_{R_{2}}, B F_{W r_{1}}\right)\right\}$

Following notations are used through the paper:
$S$ : Standby unit

O : Operative unit
$F_{r_{1}}$ : Unit is under repair which fails due to Type - I
failure
$F_{w r_{1}}$ : Unit is under waiting for repair which fails due to
Type -I failure
$F_{R_{1}}$ : Repair is continuing from previous state of Type -I
failure
$F_{r_{2}}$ : Unit is under repair which fails due to Type - II
failure
$F_{w r_{2}}$ : Unit is under waiting for repair which fails due to
Type -I failure
$F_{R_{2}}$ : Repair is continuing from previous state of Type II failure
$\lambda_{1}$ : Type - I failure rate
$\lambda_{2}$ : Type -II failure rate
$\gamma_{1}$ : Rate at which system is made operative from rest
$\gamma_{2}$ : Rate at which system is made at rest from operative state
$i(t)$ : p.d.f of time to complete pending process of material at colour sorter.
$p \quad:$ Probability that after repair unit needs not to be made operative depending upon demand
$q \quad:$ Probability that after repair unit is made operative depending upon demand
$q_{i j}(t)$ : Probability density function (p.d.f.) of first
passage time from a regenerative state $i$ to a regenerative state j or to a failed state j without visiting any other regenerative state in $(0, t]$.
$Q_{i j}(t)$ : cumulative distribution function (c.d.f) of first
passage time from a regenerative state $i$ to a regenerative state j or to a failed state j without visiting any other regenerative state in $(0, t]$.
$G_{1}(t) \quad:$ c.d.f.. of the repair time of unit for Type - I failure
$g_{1}(t):$ p.d.f. of the repair time of unit for Type - I failure
$H_{1}(t)$ : c.d.f. of time to make operative state stand by (as per demand)
$h_{1}(t)$ : p.d.f of time to make operative state stand by (as per demand)
$H_{2}(t)$ : c.d.f. of time to make stand by state operative (as per demand)
$h_{2}(t)$ : p.d.f of time to make stand by state operative (as per demand)
$p_{i j}$ : Transition probability from state ' $i$ ' to state ' $j$ '

| $p_{i j}{ }^{(k)}:$ | Transition probability from state ' $i$ ' to state ' $j$ ' |
| :--- | :--- |
|  | via state ' $k$ ' |


: unconditional mean time taken by the system to transit for any regenerative state ' $j$ ', when it is counted from the epoch of entrance into state ' $i$ ' via state ' $k$ '


Fig. 1 - Transition diagram of the two - unit system

## 3. STATISTICAL MODEL OF TWO - UNIT SYSTEMS

### 3.1 Transition Probabilities

A transition diagram shown in Fig. 1 exhibits the various states of the system. The epochs of entry into states $0,1,2,3,4$ and 5 are regenerative points. Following the approach of Goel (1981), Kumar (1980), Taneja (2001) and Tuteja (1992) the transition probabilities $p_{i j}$ can be obtained using the following formula.

$$
\begin{align*}
& p_{i j}=\lim _{s \rightarrow 0} q_{i j}{ }^{*}(s)=\lim _{s \rightarrow 0} \int_{0}^{\infty} e^{-s t} q_{i j}(t) d t \\
& \text { where, } \quad \frac{d}{d t} Q_{i j}(t)=q_{i j} \tag{1}
\end{align*}
$$

From the transition diagram, the assumptions discussed in preceding section and using equation (1), transition probabilities can be obtained as follows. The following particular cases are
considered:

$$
\mathrm{g}_{1}(\mathrm{t})=\gamma \mathrm{e}^{-\gamma t} ; \mathrm{g}_{2}(\mathrm{t})=\alpha_{1} \mathrm{e}^{-\alpha_{1} \mathrm{t}} ; \mathrm{h}_{1}(\mathrm{t})=\alpha \mathrm{e}^{-\alpha \mathrm{t}} ; \mathrm{h}_{2}(\mathrm{t})=\beta \mathrm{e}^{-\beta \mathrm{t}} .
$$

$$
\begin{aligned}
& p_{01}=h_{1}^{*}\left(\gamma_{1}+2 \lambda_{1}+2 \lambda_{2}\right) \\
& p_{02}=\frac{\gamma_{1}}{\gamma_{1}+2 \lambda_{1}+2 \lambda_{2}}\left(1-h_{1}^{*}\left(\gamma_{1}+2 \lambda_{1}+2 \lambda_{2}\right)\right) \\
& p_{03}=\frac{2 \lambda_{1}}{\gamma_{1}+2 \lambda_{1}+2 \lambda_{2}}\left(1-h_{1}^{*}\left(\gamma_{1}+2 \lambda_{1}+2 \lambda_{2}\right)\right) \\
& p_{04}=\frac{2 \lambda_{2}}{\gamma_{1}+2 \lambda_{1}+2 \lambda_{2}}\left(1-h_{1}^{*}\left(\gamma_{1}+2 \lambda_{1}+2 \lambda_{2}\right)\right) \\
& p_{10}=h_{2}^{*}\left(\gamma_{1}+\lambda_{1}+\lambda_{2}\right) \\
& p_{12}=\frac{\gamma_{1}}{\gamma_{1}+\lambda_{1}+\lambda_{2}}\left(1-h_{2}^{*}\left(\gamma_{1}+\lambda_{1}+\lambda_{2}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& p_{13}=\frac{\lambda_{1}}{\gamma_{1}+\lambda_{1}+\lambda_{2}}\left(1-h_{2}{ }^{*}\left(\gamma_{1}+\lambda_{1}+\lambda_{2}\right)\right) \\
& p_{14}=\frac{\lambda_{2}}{\gamma_{1}+\lambda_{1}+\lambda_{2}}\left(1-h_{2}{ }^{*}\left(\gamma_{1}+\lambda_{1}+\lambda_{2}\right)\right) \\
& p_{20}=1 \\
& p_{30}=q g_{1}{ }^{*}\left(\lambda_{1}+\lambda_{2}\right) \\
& p_{35}=p g_{1}{ }^{*}\left(\lambda_{1}+\lambda_{2}\right) \\
& p_{36}=\frac{\lambda_{1}}{\lambda_{1}+\lambda_{2}} g_{1}{ }^{*}\left(\lambda_{1}+\lambda_{2}\right) \\
& p_{37}=\frac{\lambda_{2}}{\lambda_{1}+\lambda_{2}} g_{1}{ }^{*}\left(\lambda_{1}+\lambda_{2}\right) \\
& p_{33}{ }^{(6)}=1-g_{1}{ }^{*}\left(\lambda_{1}\right) \\
& p_{34}{ }^{(7)}=1-g_{1}{ }^{*}\left(\lambda_{2}\right) \\
& p_{40}=q g_{2}{ }^{*}\left(\lambda_{1}+\lambda_{2}\right) \\
& p_{41}=p g_{2}{ }^{*}\left(\lambda_{1}+\lambda_{2}\right) \\
& p_{48}=\frac{\lambda_{2}}{\lambda_{1}+\lambda_{2}}\left(1-g_{2}{ }^{*}\left(\lambda_{1}+\lambda_{2}\right)\right) \\
& p_{49}=\frac{\lambda_{1}}{\lambda_{1}+\lambda_{2}}\left(1-g_{2}{ }^{*}\left(\lambda_{1}+\lambda_{2}\right)\right) \\
& p_{44}{ }^{(8)}=1-g_{2}{ }^{*}\left(\lambda_{2}\right) \\
& p_{49}{ }^{(3)}=1-g_{2}{ }^{*}\left(\lambda_{1}\right) \\
& p_{51}=1
\end{aligned}
$$

By using above equations, it can be verified that

$$
\begin{aligned}
& p_{01}+p_{02}+p_{03}+p_{04}=1 \\
& p_{10}+p_{12}+p_{13}+p_{14}=1 \\
& p_{20}=1 \\
& p_{30}+p_{35}+p_{36}+p_{37}=1 \\
& p_{40}+p_{41}+p_{48}+p_{49}=1 \\
& p_{51}=1 \\
& p_{30}+p_{35}+p_{33}{ }^{(6)}+p_{34}{ }^{(7)}=1 \\
& p_{40}+p_{41}+p_{44}{ }^{(8)}+p_{43}{ }^{(9)}=1
\end{aligned}
$$

The mean sojourn time $\mu_{i}$ in the $i^{\text {th }}$ regenerative state is defined as the time to stay in that state before transition to any other state. If T denotes the sojourn time in the regenerative state ' $i$ ', then

$$
\mu_{i}=E(t)=P_{r}(T>t)=\int_{0}^{\infty} d\left(Q_{i j}(t)\right)
$$

Eq. 2

$$
\begin{aligned}
& \mu_{0}=\frac{1}{\gamma_{1}+2 \lambda_{1}+2 \lambda_{2}}\left(1-h_{1}^{*}\left(\gamma_{1}+2 \lambda_{1}+2 \lambda_{2}\right)\right) \\
& \mu_{1}=\frac{1}{\gamma_{1}+\lambda_{1}+\lambda_{2}}\left(1-h_{2}^{*}\left(\gamma_{1}+\lambda_{1}+\lambda_{2}\right)\right) \\
& \mu_{2}=\frac{1}{\gamma_{2}} \\
& \mu_{3}=\frac{1}{\lambda_{1}}\left(1-g_{1}^{*}\left(\lambda_{1}\right)\right) \\
& \mu_{4}=\frac{1}{\lambda_{2}}\left(1-g_{2}^{*}\left(\lambda_{2}\right)\right) \\
& \mu_{5}=-i^{*^{*}}(0)=1
\end{aligned}
$$

The unconditional mean time taken by the system to transit for any regenerative state ' $j$ ', when it is counted from the epoch of entrance into state ' $i$ ' is mathematically stated as

$$
\begin{equation*}
m_{i j}=\int_{0}^{\infty} t d Q_{i j}(t)=\int_{0}^{\infty} t q_{i j}(t) d t=-q_{i j}^{*}(0) \tag{3}
\end{equation*}
$$

Thus, we get

$$
\begin{aligned}
& m_{01}+m_{02}+m_{03}+m_{04}=\mu_{0} \\
& m_{10}+m_{12}+m_{13}+m_{14}=\mu_{1} \\
& m_{20}=\mu_{2} \\
& m_{30}+m_{35}+m_{36}+m_{37}=\mu_{3} \\
& m_{40}+m_{41}+m_{48}+m_{49}=\mu_{4} \\
& m_{51}=\mu_{5} \\
& m_{30}+m_{35}+m_{33}{ }^{(6)}+m_{34}{ }^{(7)}=k_{3}(\text { say }) \\
& m_{40}+m_{41}+m_{44}{ }^{(8)}+m_{43}{ }^{(9)}=k_{4}
\end{aligned}
$$

### 3.2. Mean Time to System Failure (MTSF)

Let $\phi_{i}(t)$ be the c.d.f. of the first passage time from regenerative state $i$ to a failed state. In order to determine the mean time to system failure (MTSF) of the system, considering the failed state as absorbing states. Following the approach of Goel (1981), Kumar (1980), Taneja (2001) and Tuteja (1992), we obtain the following recursive relation for $\phi_{i}(t)$ :
$\phi_{0}(t)=Q_{01}(t) \odot \phi_{1}(t)+Q_{02}(t) \odot \phi_{2}(t)+Q_{03}(t) \odot \phi_{3}(t)+Q_{04}(t) \odot \phi_{4}(t)$
$\phi_{1}(t)=Q_{10}(t) \odot \phi_{1}(t)+Q_{12}(t) \odot \phi_{2}(t)+Q_{13}(t) \odot \phi_{3}(t)+Q_{14}(t) \odot \phi_{4}(t)$
$\phi_{2}(t)=Q_{20}(t) \odot \phi_{0}(t)$

$$
\begin{align*}
& \phi_{3}(t)=Q_{30}(t) \odot \phi_{0}(t)+Q_{35}(t) \odot \phi_{5}(t)+Q_{36}(t)+Q_{37}(t) \\
& \phi_{4}(t)=Q_{41}(t) \odot \phi_{1}(t)+Q_{42}(t) \odot \phi_{2}(t)+Q_{48}(t)+Q_{49}(t) \\
& \phi_{5}(t)=Q_{51}(t) \odot \phi_{1}(t) \tag{4}
\end{align*}
$$

### 3.3. Availability

Let $A_{i}(t)$ be the probability that the system is in up state at instant $t$ given that the system entered regenerative state $i$ at $t=0$. Following the method used in section 3.2, the availability $A_{i}(t)$ is expressed as the following recursive relations:

$$
\begin{aligned}
A_{0}(t)= & M_{0}(t)+q_{01}(t) ® A_{1}(t)+q_{02}(t) \circlearrowleft A_{2}(t)+q_{03}(t) ® A_{3}(t) \\
& +q_{04}(t) ® A_{4}(t)
\end{aligned}
$$

$$
A_{1}(t)=M_{1}(t)+q_{10}(t) \odot A_{1}(t)+q_{12}(t) \circlearrowleft A_{2}(t)+q_{13}(t) \circlearrowleft A_{3}(t)
$$

$$
+q_{14}(t) \Subset A_{4}(t)
$$

$A_{2}(t)=q_{20}(t) \odot A_{0}(t)$
$A_{3}(t)=M_{3}(t)+q_{30}(t) ® A_{0}(t)+q_{35}(t) ® A_{5}(t)+$
$q_{33}{ }^{(6)}(t) \odot A_{3}(t)+q_{34}{ }^{(7)}(t) \odot A_{4}(t)$
$A_{4}(t)=q_{41}(t) \odot A_{1}(t)+q_{42}(t) ® A_{2}(t)+q_{44}{ }^{(8)}(t) \odot A_{4}(t)$
$+q_{43}{ }^{(9)}(t) \odot A_{3}(t)$
$A_{5}(t)=M_{5}(t)+q_{51}(t) ® A_{1}(t)$
Eq. (5)
where $M_{i}(t)$ is the probability that the system is up at time $t$ without any transition through/to any other regenerative state or returning to itself through one or more non-regenerative states. Thus,

$$
\begin{aligned}
& M_{0}(t)=e^{-\left(2 \lambda_{1}+2 \lambda_{2}\right) t} \bar{H}_{1}(t) \\
& M_{1}(t)=e^{-\left(\gamma_{1}+\lambda_{1}+\lambda_{2}\right) t} \bar{H}_{2}(t) \\
& M_{3}(t)=e^{-\left(\lambda_{1}+\lambda_{2}\right) t} \bar{G}_{2}(t) \\
& M_{4}(t)=e^{-\left(\lambda_{1}+\lambda_{2}\right) t} \bar{G}_{1}(t) \\
& M_{5}(t)=\bar{I}(t)
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{G}_{1}(t)= & 1-G_{1}(t), \bar{H}_{1}(t)=1-H_{1}(t), \bar{G}_{2}(t)=1-G_{2}(t), \\
& \bar{H}_{2}(t)=1-H_{2}(t)
\end{aligned}
$$

### 3.4. Busy period of repairman

Let $B_{i}(t)$ be the probability that a system, having started from regenerative state $S_{i}(i=0,1 \ldots .9)$ at $t=0$, is under the services of repairman. Following the method used in section 3.2, we
have
$B_{0}(t)=q_{01}(t) \odot B_{1}(t)+q_{02}(t) \odot B_{2}(t)+q_{03}(t) \odot B_{3}(t)+q_{04}(t) \odot B_{4}(t)$
$B_{1}(t)=q_{10}(t) \odot B_{1}(t)+q_{12}(t) \odot B_{2}(t)+q_{13}(t) \odot B_{3}(t)+q_{14}(t) \odot B_{4}(t)$
$B_{2}(t)=q_{20}(t) \odot B_{0}(t)$
$B_{3}(t)=W_{3}(t)+q_{30}(t) \circlearrowleft B_{0}(t)+q_{35}(t) \circlearrowleft B_{5}(t)+q_{33}{ }^{(6)}(t) \odot B_{3}(t)+q_{34}{ }^{(7)}(t) \circlearrowleft B_{4}(t)$
$B_{4}(t)=W_{4}(t)+q_{41}(t) \odot B_{1}(t)+q_{42}(t) \odot B_{2}(t)+q_{44}{ }^{(8)}(t) \circlearrowleft B_{4}(t)+q_{43}{ }^{(9)}(t) \odot B_{3}(t)$ $B_{5}(t)=q_{51}(t) \bigcirc B_{1}(t)$

Eq. (6)

## 4. COMPUTATION OF RELIABILITY METRICS

In order to obtain MTSF, we first take Laplace transforms of equations (4) and then solve them for $\phi_{0}{ }^{* *}(s)$. Thus, we get

$$
\begin{equation*}
M T S F=\lim _{s \rightarrow 0} \frac{1-\varphi_{0}{ }^{* *}(s)}{s}=\frac{N}{D} \tag{7}
\end{equation*}
$$

Where

$$
\begin{aligned}
N= & \mu_{0}\left(-p_{14} p_{41}-p_{13} p_{35}+1\right)-\mu_{1}\left(-p_{01}-p_{04} p_{41}-p_{03} p_{35}\right) \\
& -\mu_{2}\left(-p_{02} p_{12}-p_{04} p_{12} p_{41}-p_{12} p_{03} p_{35}-p_{02}+p_{02} p_{14} p_{41}\right) \\
& -\mu_{3}\left(-p_{03}+p_{03} p_{14} p_{41}-p_{01} p_{13}-p_{04} p_{13} p_{41}\right) \\
& -\mu_{4}\left(-p_{04}+p_{04} p_{13} p_{35}-p_{01} p_{14}-p_{03} p_{14} p_{35}\right) \\
& -\mu_{5}\left(-p_{03} p_{35}-p_{01} p_{13} p_{35}-p_{13} p_{35} p_{04} p_{41}+p_{03} p_{35} p_{14} p_{41}\right.
\end{aligned}
$$

$$
\begin{aligned}
& D=1-p_{14} p_{41}-p_{13} p_{35}-p_{04} p_{40}-p_{02}+p_{02} p_{14} p_{41}+p_{02} p_{13} p \\
& \quad+p_{03} p_{30} p_{14} p_{41}-p_{01} p_{10}-p_{01} p_{14} p_{40}-p_{01} p_{13} p_{30}-p_{04} p_{41} p \\
& \quad-p_{12} p_{03} p_{35}-p_{03} p_{35} p_{10}-p_{03} p_{35} p_{14} p_{40}
\end{aligned}
$$

Next, taking Laplace transforms of equations (5) and solving them for $A_{0}^{*}(s)$, we get

$$
\begin{equation*}
A_{0}=\lim _{s \rightarrow 0} s A_{0} *(s)=\frac{N_{1}}{D_{1}} \tag{8}
\end{equation*}
$$

Where

$$
\begin{aligned}
D_{1}= & \mu_{0}\left\{p_{30}\left(1-p_{44}{ }^{(8)}\right)+p_{35} p_{40}\left(1-p_{13}\right)+\left(p_{10}+p_{12}\right)\right. \\
& \left.\left(p_{35} p_{41}+p_{35} p_{43}{ }^{(9)}+p_{34}{ }^{(7)} p_{41}\right)-p_{30} p_{14} p_{41}+p_{34}{ }^{(7)} p_{40}\right\} \\
& +\mu_{1}\left\{p_{01} p_{30}\left(1-p_{44}{ }^{(8)}\right)+p_{04} p_{30} p_{41}+p_{35} p_{40}\left(p_{01}+p_{03}\right)\right. \\
& \left.+p_{01} p_{40} p_{34}{ }^{(7)}+\left(1-p_{02}\right)\left(p_{35} p_{41}+p_{35} p_{43}{ }^{(9)}+p_{41} p_{34}{ }^{(7)}\right)\right\} \\
& +k_{3}\left\{p_{03} p_{40}+p_{03} p_{41}-p_{03} p_{14} p_{41}+p_{01} p_{40} p_{13}+\left(1-p_{02}-p_{03}\right)\right. \\
& \left.p_{13} p_{41}-p_{01}\left(p_{10}+p_{12}\right) p_{43}{ }^{(9)}+\left(1-p_{02}\right) p_{43}^{(9)}\right\}+k_{4}\left\{p_{35} p_{14}\right. \\
& \left(p_{01}+p_{03}\right)-p_{01}\left(p_{10}+p_{12}\right) p_{34}{ }^{(7)}+\left(1-p_{02}\right) p_{34}{ }^{(7)}+p_{35} p_{04} \\
& \left.\left(1-p_{13}\right)+p_{30} p_{04}+p_{01} p_{14} p_{30}\right\}+\mu_{2}\left\{p_{01} p_{12}\left(p_{30}+p_{35}+p_{34}{ }^{(7)}\right)\right. \\
& \left(p_{40}+p_{41}+p_{43}{ }^{(9)}-p_{14} p_{41}\right)-p_{01} p_{12} p_{34}{ }^{(7)} p_{43}{ }^{(9)}-p_{01} p_{12} p_{13} p_{41} p_{34}^{(7)} \\
& -p_{01} p_{12} p_{43}{ }^{(9)} p_{14} p_{35}-p_{01} p_{12} p_{13} p_{35}\left(p_{40}+p_{41}+p_{43}{ }^{(9)}\right)
\end{aligned}
$$

$$
\begin{aligned}
N_{1}= & \left(M_{0}+p_{01} M_{1}\right)\left(1-p_{44}{ }^{(8)}-p_{14} p_{41}-p_{33}{ }^{(6)}+p_{33}{ }^{(6)} p_{44}{ }^{(8)}\right. \\
& \left.+p_{33}{ }^{(6)} p_{14} p_{41}+p_{13} p_{35}+p_{13} p_{35} p_{44}^{(8)}+p_{13} p_{35} p_{14} p_{41}\right) \\
& +\left(M_{4}+p_{41} M_{1}\right)\left(p_{03} p_{34}{ }^{(7)}+p_{03} p_{14} p_{35}+p_{04}-p_{04} p_{33}{ }^{(6)}\right. \\
& \left.-p_{13} p_{35} p_{04}\right)+M_{3}\left(-p_{03} p_{44}^{(8)}+p_{03}-p_{03} p_{14} p_{41}+p_{04} p_{43}{ }^{(9)}\right. \\
& \left.+p_{04} p_{13} p_{41}\right)+p_{03} p_{35}-p_{03} p_{35} p_{44}^{(8)}+p_{03} p_{35} p_{14} p_{41} \\
& +p_{04} p_{35} p_{43}{ }^{(9)}+p_{13} p_{35} p_{04} p_{41}+\left(p_{03}+p_{01} p_{13}\right) \\
& \left(p_{12} p_{35} p_{40}+p_{12} p_{35} p_{41}+p_{12} p_{35} p_{43}{ }^{(9)}+p_{12} p_{34}{ }^{(7)} p_{41}\right) \\
& +\left(p_{04}+p_{01} p_{14}\right)\left(p_{12} p_{35} p_{43}{ }^{(9)}+p_{12} p_{30} p_{41}+p_{12} p_{35} p_{41}\right. \\
& \left.\left.+p_{12} p_{34}{ }^{(7)} p_{41}\right)\right\}+\mu_{5}\left\{\left(1-p_{02}-p_{01}\left(p_{10}+p_{12}\right)\right)\right. \\
& \left(p_{14} p_{35} p_{13} p_{41}+p_{14} p_{35} p_{43}{ }^{(9)}+p_{13} p_{35} p_{40}+p_{13} p_{35} p_{41}\right. \\
& \left.+p_{13} p_{35} p_{43}{ }^{(9)}\right)+\left(p_{03}+p_{01} p_{13}\right)\left(\left(p_{10}+p_{12}\right) p_{35} p_{40}\right. \\
& \left.+\left(p_{10}+p_{12}\right) p_{35} p_{41}+\left(p_{10}+p_{12}\right) p_{35} p_{43}{ }^{(9)}+p_{14} p_{35} p_{40}\right) \\
& \left.+\left(p_{04}+p_{01} p_{14}\right)\left(p_{10} p_{35} p_{43}{ }^{(9)}+p_{12} p_{35} p_{43}{ }^{(9)}+p_{13} p_{35} p_{40}\right)\right\}
\end{aligned}
$$

Finally, again taking Laplace transforms of equations (6) and solving them for $B_{0}{ }^{*}(s)$ we get

$$
\begin{equation*}
B_{0}{ }^{*}(s)=\frac{N_{3}(s)}{D_{1}(s)} \tag{9}
\end{equation*}
$$

In steady-state, the total fraction of time for which the system is under the service of assistant repairman is given by

$$
B_{o}=\lim _{s \rightarrow 0} B_{o}^{*}(s)=\frac{N_{3}}{D_{1}}
$$

Where

$$
\begin{aligned}
N_{3}= & \left(p_{03}+p_{04}\right)\left[\begin{array}{l}
p_{48}+p_{49}+W_{4}-p_{41} p_{14}\left(p_{48}+p_{49}+W_{4}\right) \\
-p_{13} p_{35}\left(p_{48}+p_{48}+W_{4}\right)
\end{array}\right] \\
& +\left(p_{03}+p_{04}+p_{13}+p_{14}+p_{04} p_{41}+p_{03} p_{35}\right)\left(p_{36}+p_{37}+W_{3}\right)
\end{aligned}
$$

and $D_{1}$ is already specified.
One of the objectives of reliability analysis is to optimize the profit incurred to the system. To achieve this, profit model is defined by subtracting all expected maintenance liabilities from the total revenue. Using equations (8) and (9), we get

$$
P=C_{0} A_{0}-C_{1} B_{0}
$$

where
$C_{0}=$ Total revenue per unit time and $C_{1}=$ cost of busy period of repairman.

On the basis of these data, we have computed the following rates: $\mathrm{g}_{1}(\mathrm{t})=0.01, \mathrm{~g}_{2}(\mathrm{t})=0.0123, \gamma_{1}=0.01736$ and $\gamma_{2}=0.0987$. Assuming $\mathrm{h}_{1}(\mathrm{t})=0.01, \mathrm{~h}_{2}(\mathrm{t})=0.01$ and $\mathrm{p}=0.8$, we have computed the following results of important reliability indices using the software 'MATLAB'. By varying $\lambda_{2}$ for different values of $\lambda_{1}$, the values of MTSF and availability are computed and their behaviors are exhibited in graphs (figure 2 and figure 3). Similarly, the profit is also computed by varying $C_{0}$ for different choices of $C_{1}$ and results are presented in figure 4.

Fig. 2 shows the behavior of MTSF with respect to Type-II failure rate $\left(\lambda_{2}\right)$ for different values of Type-I failure rate $\left(\lambda_{1}\right)$. The graph shows that MTSF decreases with increase in the Type - II failure rate $\left(\lambda_{2}\right)$ keeping Type - I failure rate constant and has higher values for lower values of Type - I failure rate $\left(\lambda_{1}\right)$. Fig. 3 shows the behavior of availability with respect to Type II failure rate $\left(\lambda_{2}\right)$ for different values of Type - I failure rate $\left(\lambda_{1}\right)$. This graph indicates that availability of the system decreases with increase in the Type - II failure rate $\left(\lambda_{2}\right)$ keeping Type - I failure rate constant and has higher values for lower values of Type - I failure rate $\left(\lambda_{1}\right)$. So, the management of the manufacturing plant should pay more attention on the working of colour sorter part for increasing the MTSF as well as availability. The behavior of profit with respect to revenue $\left(C_{0}\right)$ for different values of cost of repairman $\left(C_{1}\right)$ is shown in Fig. 4. It is observed from this graph that profit decreases with the increase in revenue per unit time $C_{0}$ and has higher values for lower values of cost of repairman $C_{1}$.

On comparing the graphs, it reveals that (i) for $C_{1}=$ 850, the profit is negative or zero or positive according as $C_{0} \leq$ or $\geq 1491.50$. Hence, revenue per unit time should be fixed greater than 1491.5 and (ii) for $C_{1}=900$, the profit is positive or zero or negative according as $C_{0} \leq$ or $\geq 1564$. Hence, revenue per unit time should be fixed greater than 1564 (iii) For $\mathrm{C} 1=950$, the profit is positive or zero or negative according as $C_{0} \leq$ or $\geq 1658.5$. Hence, revenue per unit time should be fixed greater than 1658.5

The present analysis provides important information about the sensitivity of the particular components of the standby systems which need more care to achieve the maximum profit. This model can be applied to any industry having two unit standby systems. The same approach can be extended to study those industrial systems which have more than two unit stand by systems.

## 4. CONCLUSIONS AND DISCUSSIONS

In this study data for all types of failures and repairs of the concerned industry was collected in the units of per hour.


Fig. 2: Effect of Type-II failure rate on Mean time to system failure for different values of Type-I failure rate.


Fig. 3 : Effect of Type-II failure rate on availability for different values of Type-I failure rate


Fig. 4 : Effect of cost of revenue per unit time on Profit achieved by the system for different values of cost of busy period of repairman.

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